3-D Topology Optimization of Dielectric Resonator in Waveguide Structure Considering Higher Mode Incidence

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Abstract — In this paper, a 3-D topology optimization of dielectric material in waveguide structure is presented. It is based on material sensitivity formulation in the high frequency domain. To obtain the dielectric structures that do not span the entire height of the waveguide, higher mode incidence waves at the waveguide ports are considered. Also, issues that are encountered when implementing 3-D topology optimization with commercial EM software are explained, and their solutions are presented. Also, the duality of the E-field and H-field formulations of the material sensitivity is explored and summarized.

I. Introduction

Topology optimization is the optimal design method which is based on the universal, gray-scale pixel-like representation of the device. It has several merits compared to the conventional shape optimization method, such as fully admissible design space, and the ability to obtain an original design of the device. This method originated from and is most popular in structural mechanics, but there have been considerable works for the low-frequency electrical machines as well [1]. Recently, a number of papers have appeared that deals with topology optimization of high frequency systems [2]-[3]. These works are based on the adjoint variable method (AVM) and material sensitivity formulation that allows fast and efficient calculation of sensitivity information. This is especially important for high resolution topology optimization, where there are a large number of design variables. In such design problems, it is computationally prohibitive to use stochastic design methods or simple calculation of sensitivity based on finite difference.

However, in the previous papers, the demonstrated numerical examples for topology optimization were all simple 2-D structures, because a custom EM solver was used in the analysis module. One of the advantages of the material sensitivity formulation given in [3] is that it does not depend on any specific analysis technique. Thus, it is possible to use the commercial EM software such as Ansoft HFSS to get the field solution at design grids and calculate sensitivity from the field values. Using commercial EM software, it is also easier to expand the optimization to 3-D problems and apply complex higher mode boundary conditions at the ports.

In this paper, we apply the material continuum sensitivity formula to the 3-D topology optimization of dielectric resonators in waveguide structure. To obtain dielectric structures that do not span the entire height of the waveguide, higher mode incidence waves at the waveguide ports are considered. Also, practical issues that are encountered when commercial EM software is combined with topology optimization are explained, including the process of assigning different material parameters at each design grid.

II. Dual Sensitivity Formulations

In this chapter, the material sensitivity formula from [3] is summarized, and duality of **E**-field and **H**-field formulations is investigated. An objective function can be defined on port boundary Γ_1 as

minimize

$$O = \int_{\Gamma_1} g\left(\mathbf{H}(\mathbf{p})\right) d\Gamma \tag{1}$$

subject to

$$\nabla \times \left(\varepsilon_r^{-1} \nabla \times \mathbf{H}(\mathbf{p})\right) - k_0^2 \mu_r \mathbf{H}(\mathbf{p}) = 0 \tag{2}$$

where g is a scalar function, and the system parameter vector $\mathbf{p}$ controls permittivity ε or permeability μ distribution of the system. On Γ_1 where incident field is imposed, boundary condition of the third kind should be defined, which may be expressed as

$$\varepsilon_r^{-1} \hat{n} \times \left(\nabla \times \mathbf{H}\right) + \gamma_h \hat{n} \times (\hat{n} \times \mathbf{H}) = \mathbf{V} \tag{3}$$

where γ_h is a known parameter and $\mathbf{V}$ is a known vector that has a magnitude proportional to that of the incident field. The material sensitivity formula is derived based on augmented Lagrangian method, and is written by

$$\frac{d\bar{O}}{d\mathbf{p}} = \int_\Omega \left[\frac{\partial(\varepsilon_r^{-1})}{\partial \mathbf{p}} \left(\nabla \times \mathbf{H}\right) \cdot \left(\nabla \times \boldsymbol{\lambda}(\mathbf{H})\right) - k_0^2 \frac{\partial \mu_r}{\partial \mathbf{p}} \mathbf{H} \cdot \boldsymbol{\lambda}(\mathbf{H}) \right] d\Omega \tag{4}$$

where adjoint variable vector $\boldsymbol{\lambda}(\mathbf{H})$ is given by the following **H**-field adjoint equation, given in a variational form as,

$$\int_\Omega \left[-\varepsilon_r^{-1} \left(\nabla \times \bar{\boldsymbol{\lambda}}\right) \cdot \left(\nabla \times \boldsymbol{\lambda}\right) + k_0^2 \mu_r \bar{\boldsymbol{\lambda}} \cdot \boldsymbol{\lambda} \right] d\Omega - \int_{\Gamma_1} \left[\gamma_h \left(\hat{n} \times \bar{\boldsymbol{\lambda}}\right) \cdot \left(\hat{n} \times \boldsymbol{\lambda}\right) + \bar{\boldsymbol{\lambda}} \cdot \mathbf{g_H} \right] d\Gamma = 0 \tag{5}$$

where $\bar{\boldsymbol{\lambda}}$ can be an arbitrary variable, and $\mathbf{g_H} \equiv \partial g / \partial \mathbf{H}$ is adjoint source. When the objective function is given in terms of electric field, the **E**-field material sensitivity formulation can be derived using the similar procedure as

$$\frac{d\bar{O}}{d\mathbf{p}} = \int_\Omega \left[\frac{\partial(\mu_r^{-1})}{\partial \mathbf{p}} \left(\nabla \times \mathbf{E}\right) \cdot \left(\nabla \times \boldsymbol{\lambda}(\mathbf{E})\right) - k_0^2 \frac{\partial \varepsilon_r}{\partial \mathbf{p}} \mathbf{E} \cdot \boldsymbol{\lambda}(\mathbf{E}) \right] d\Omega \tag{6}$$

where $\boldsymbol{\lambda}(\mathbf{E})$ is adjoint variable derived from **E**-field adjoint equation. The **H**-field and **E**-field material sensitivity represented by (4) and (6) form dual set of equations. Either formulation can be used to calculate sensitivity. However, for easier implementation, the choice of the sensitivity formula should be made based upon the form of the objective function O, and the type of the reference incident field (**H**-field or **E**-field).

III. Application to 3-D Waveguide Problem

For the topology optimization of 3-D waveguide, the objective function is given in the following equation.

$$\text{minimize} \quad O = \frac{1}{n}\sum_{i=1}^{n}\left|T_i - T_0\right|^2 \tag{7}$$

where n is total frequency points, T_i is the transmission coefficient calculated at the ith frequency, and T_o is the desired value at each frequency. It should be noted that this objective function involves the phase information of the transmission coefficient as well as its magnitude. The adjoint source term can be derived as

$$\frac{\partial O}{\partial \mathbf{E}} = \frac{1}{n}\sum_{i=1}^{n}\frac{\partial\left|T_i - T_0\right|^2}{\partial \mathbf{E}} = \frac{1}{n}\sum_{i=1}^{n}\frac{1}{\mathbf{E}_{inc}}(T_i - T_0)^* \tag{8}$$

where $\mathbf{E}_{inc}$ is the incident electric field at each frequency. Since the objective function and adjoint source is given in terms of electric field, (6) should be used for sensitivity calculation. For the optimal distribution of dielectric material, the first term in the integral of (6) vanishes since $\partial(\mu_r^{-1})/\partial\mathbf{p} = 0$. To define relationship between relative permittivity ε_r and design parameter, a modified version of the density method is used [3]:

$$\varepsilon_r = \varepsilon_{r\min}\left(\varepsilon_{r\max}/\varepsilon_{r\min}\right)^p \tag{9}$$

where p is the normalized density of the unit voxel ($0 \le p \le 1$), and $\varepsilon_{r\min}$ and $\varepsilon_{r\max}$ are the minimum and maximum value of the relative permittivity distribution in the design region, respectively. Assuming the unit voxel is sufficiently small, and setting $\varepsilon_{r\min} = 0$ to represent empty space in the waveguide, the mth component of the sensitivity vector can be written as

$$d\bar{O}/dp_m = -V_m k_0^2 \varepsilon_{\max}^{p_m} \ln(\varepsilon_{r\max}) \times \left(\mathbf{E}\cdot\boldsymbol{\lambda}(\mathbf{E})\right) \tag{10}$$

where V_m and p_m are the volume and normalized density of the mth voxel, respectively, and **E** and **λ**(**E**) denote average field values taken over the voxel.

Care must be taken when the adjoint source given by (8) is applied to commercial EM software such as Ansoft HFSS. In HFSS, the waveguide port source is expressed in terms of input power. However, the incident sources of the primary and adjoint system were derived in terms of **H**-field or **E**-field in the preceding formulations. Thus, the input power of the port should be adjusted to obtain the correct incident fields at the ports.

IV. Optimization Results and Discussion

Fig. 1 shows the shape of the 3-D waveguide structure to be optimized. The design goal is to obtain the desired transmission coefficient at 2.4 GHz, and the design variables are set as the relative permittivity parameters in the three regions depicted in Fig. 1. The bottom and right side is set as symmetry boundary and a quarter of the full model is solved. The convergence of the objective function is shown in Fig. 2, and the change of design variables and objective function before and after the optimization is given in Table I. Initially, the whole waveguide is set as empty space. However, as optimization process goes on, left and right grid is filled with dielectric material, while center grid remains empty. The exact target value of T was reached in 27 iterations, which confirms the validity of the proposed method.

In the extended paper, to obtain more complex dielectric structure that does not span the entire cross-section of the waveguide, smaller voxels will be generated and higher mode incidence waves at the waveguide ports will be considered.

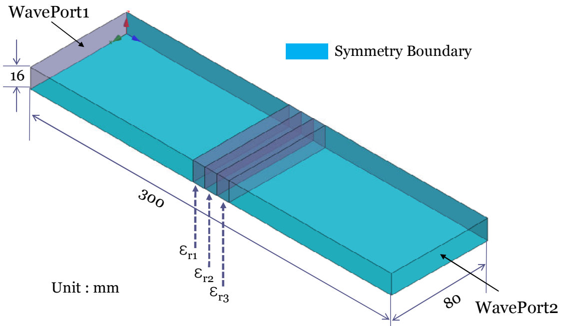

Fig. 1. The waveguide structure and design variables.

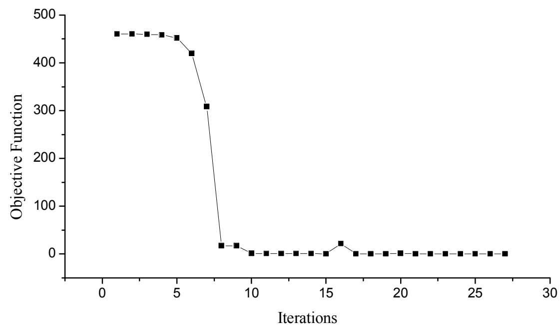

Fig. 2. Convergence of the objective function.

TABLE I
Comparison of Design Variables and Objective Function

Variables	ε_{r1}	ε_{r2}	ε_{r3}	F	T	T_0
Initial	1	1	1	460.17	1	0.3593
Optimal	9.9967	1.0092	9.9967	0	0.3593	0.3593

Design variables values are relative permittivity of the 3 cells shown in Fig. 1. F is the objective function. T and T_0 are calculated transmission coefficient and target value, respectively.

V. References

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